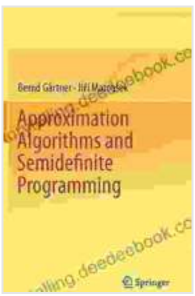
Approximation Algorithms And Semidefinite Programming: A Comprehensive Overview

Approximation algorithms are algorithms that find approximate solutions to NP-hard problems. NP-hard problems are problems that are difficult to solve exactly, and approximation algorithms provide a way to find solutions that are close to the optimal solution in polynomial time.



Approximation Algorithms and Semidefinite Programming by Bernd Gärtner

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Semidefinite programming (SDP) is a type of optimization problem that can be used to solve approximation algorithms. SDP problems are solved by finding the minimum or maximum of a linear function subject to a set of linear constraints.

The combination of approximation algorithms and SDP has been used to solve a wide variety of NP-hard problems, including:

- The maximum cut problem
- The traveling salesman problem

- The graph coloring problem
- The minimum vertex cover problem
- The maximum independent set problem

In this article, we will provide a comprehensive overview of approximation algorithms and SDP. We will discuss the basic concepts of approximation algorithms and SDP, and we will show how these techniques can be used to solve NP-hard problems.

Approximation Algorithms

Approximation algorithms are algorithms that find approximate solutions to NP-hard problems. NP-hard problems are problems that are difficult to solve exactly, and approximation algorithms provide a way to find solutions that are close to the optimal solution in polynomial time.

The quality of an approximation algorithm is measured by its approximation ratio. The approximation ratio is the ratio of the cost of the solution found by the algorithm to the cost of the optimal solution. The lower the approximation ratio, the better the algorithm.

There are a number of different techniques that can be used to design approximation algorithms. Some of the most common techniques include:

- **Greedy algorithms**: Greedy algorithms make a series of locally optimal choices in the hope of finding a globally optimal solution. Greedy algorithms are often simple to implement, but they can sometimes produce poor approximations.

- **Randomized algorithms**: Randomized algorithms use randomness to make their decisions. Randomized algorithms can often produce better approximations than greedy algorithms, but they are more difficult to analyze.
- **Linear programming relaxations**: Linear programming relaxations are a way to relax an NP-hard problem into a linear programming problem. Linear programming problems can be solved in polynomial time, and the solution to the relaxed problem can be used to find an approximation to the solution of the original problem.
- **Semidefinite programming relaxations**: Semidefinite programming relaxations are a generalization of linear programming relaxations. SDP relaxations can be used to find approximations to a wider range of NP-hard problems than linear programming relaxations.

Semidefinite Programming

Semidefinite programming (SDP) is a type of optimization problem that can be used to solve approximation algorithms. SDP problems are solved by finding the minimum or maximum of a linear function subject to a set of linear constraints.

The variables in an SDP problem are matrices. The matrices must be positive semidefinite, which means that they must have nonnegative eigenvalues.

SDP problems can be solved using a variety of methods, including:

- **Interior-point methods**: Interior-point methods are a class of algorithms that find the solution to an SDP problem by moving from the interior of the feasible region to the boundary.

- **Cutting-plane methods**: Cutting-plane methods are a class of algorithms that find the solution to an SDP problem by adding cutting planes to the feasible region.

Approximation Algorithms And SDP

The combination of approximation algorithms and SDP has been used to solve a wide variety of NP-hard problems. SDP relaxations can be used to find approximations to a wider range of NP-hard problems than linear programming relaxations.

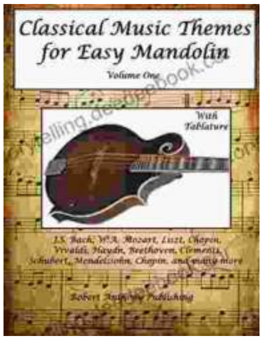
Some of the most successful applications of approximation algorithms and SDP include:

- **The maximum cut problem**: The maximum cut problem is a problem that asks for the maximum cut of a graph. A cut is a set of edges that, when removed from the graph, split the graph into two disjoint sets of vertices. The maximum cut problem is NP-hard, but it can be approximated to within a factor of 0.5 using an SDP relaxation.
- **The traveling salesman problem**: The traveling salesman problem is a problem that asks for the shortest tour of a set of cities. The traveling salesman problem is NP-hard, but it can be approximated to within a factor of 2 using an SDP relaxation.
- **The graph coloring problem**: The graph coloring problem is a problem that asks for the minimum number of colors needed to color the vertices of a graph so that no two adjacent vertices have the same color. The graph coloring problem is NP-hard, but it can be approximated to within a factor of 3 using an SDP relaxation.

- **The minimum vertex cover problem**: The minimum vertex cover problem is a problem that asks for the minimum set of vertices that cover all of the edges of a graph. The minimum vertex cover problem is NP-hard, but it can be approximated to within a factor of 2 using an SDP relaxation.
- **The maximum independent set problem**: The maximum independent set problem is a problem that asks for the maximum set of vertices that are not adjacent to each other in a graph. The maximum independent set problem is NP-hard, but it can be approximated to within a factor of 2 using an SDP relaxation.

Approximation algorithms and SDP are powerful tools for solving NP-hard problems. The combination of these two techniques has been used to solve a wide variety of problems, including the maximum cut problem, the traveling salesman problem, the graph coloring problem, the minimum vertex cover problem, and the maximum independent set problem.

As the field of approximation algorithms and SDP continues to develop, we can expect to see even more applications of these techniques to solve important problems in a variety of fields.



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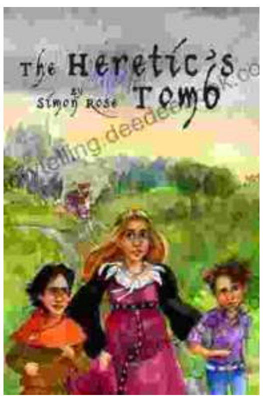

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